Chapter 9. Introduction to Trigonometry

त्रिकोणमितिय अनुपातों को निम्न अनुसार बदले जैसे कि नीचे निर्देश दिया गया है | जैसे sinθ से cosθ में बदला जा सकता है अथवा sinθ से cosecθ में बदला जा सकता है |

नियम:

$\sin\theta \Rightarrow \cos\theta \Rightarrow \sec\theta \Rightarrow \tan\theta \Rightarrow \cot\theta \Rightarrow \operatorname{cosec}\theta \Rightarrow \sin\theta$

अथवा (Or)

$\sin\theta \Leftarrow \cos\theta \Leftarrow \sec\theta \Leftarrow \tan\theta \Leftarrow \cot\theta \Leftarrow \operatorname{cosec}\theta \Leftarrow \sin\theta$

(i) जैसे यदि cosθ में बदलना हो तो $\sin^2\theta = 1 - \cos^2\theta$ का प्रयोग करेंगे यदि cosecθ की तरफ जाना हो तो $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$ का प्रयोग होगा |

(ii) अब cosθ से secθ में बदलना है तो $\cos\theta = \frac{1}{\sec\theta}$ का प्रयोग होगा या cosθ से sinθ की ओर बदलना है तो $\cos^2\theta = 1 - \sin^2\theta$ का पप्रयोग होगा |

(iii) अब secθ से tanθ की ओर बदलना है तो $\sec^2\theta = 1 + \tan^2\theta$ का प्रयोग होगा | यदि secθ से cosθ में बदलना है तो $\sec\theta = \frac{1}{\cos\theta}$ का प्रयोग होगा |

(iv) यदि tanθ से cotθ में बदलना है तो व्युत्क्रम $\tan\theta = \frac{1}{\cot\theta}$ या cotθ से tanθ में बदलना है तो $\cot\theta = \frac{1}{\tan\theta}$ का प्रयोग करेंगे |

(v) cotθ से cosecθ में बदलना है तो $\cot^2\theta = \operatorname{cosec}^2\theta - 1$ का प्रयोग किया जायेगा और cosecθ से cotθ में बदलना है तो $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ का प्रयोग किया जायेगा |

(vi) किसी भी त्रिकोणमितिय अनुपात को sinθ या cosθ में बदलकर हल करना सुविधाजनक होता है |

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Exercise 8.4 (NCERT)

Q1. त्रिकोणमितिय अनुपातों sin A, sec A और tan A को cot A के पदों में व्यक्त कीजिए |

Solution:

$\sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{\sqrt{\cot^2 A + 1}}$ $[\because \operatorname{cosec}\theta = \sqrt{\cot^2\theta + 1}]$

$\sec A = \sqrt{\tan^2 A + 1}$ $\left[\because \sec\theta = \sqrt{\tan^2\theta + 1}\right]$

$= \sqrt{\frac{1}{\cot^2 A} + 1}$

$= \sqrt{\frac{1 + \cot^2 A}{\cot^2 A}}$ $\left[\because \sec\theta = \sqrt{\tan^2\theta + 1}\right]$

$\tan A = \frac{1}{\cot A}$

Q2. ∠A के अन्य सभी त्रिकोणमितिय अनुपातों को secA के पदों में व्यक्त कीजिए |

Solution:

$\sin A = \sqrt{1 - \cos^2 A}$

$= \sqrt{1 - \frac{1}{\sec^2 A}}$

$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}$

$= \frac{\sqrt{\sec^2 A - 1}}{\sec A}$

$\cos A = \frac{1}{\sec A}$

$\tan A = \sqrt{\sec^2 A - 1}$

$\operatorname{cosec} A = \frac{1}{\sin A}$

$= \frac{1}{\sqrt{1 - \cos^2 A}}$

$= \frac{1}{\sqrt{1 - \frac{1}{\sec^2 A}}}$

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$$= \frac{1}{\sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}}$$

$$= \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$= \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{sec^2 A - 1}}$$

Q3. मान निकालिए |

(i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

Solution:

$$\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ} = \frac{\sin^2 63^\circ + \cos^2 (90^\circ - 27^\circ)}{\sin^2 (90^\circ - 17^\circ) + \cos^2 73^\circ}$$

$$= \frac{\sin^2 63^\circ + \cos^2 63^\circ}{\sin^2 73^\circ + \cos^2 73^\circ} = \frac{1}{1} = 1$$

$\sin\theta = \cos(90 - \theta)$
$\cos\theta = \sin(90 - \theta)$

(ii) sin 25° cos 65° + cos 25° sin 65°

Solution:

sin 25° cos 65° + cos 25° sin 65°

= sin 25° sin(90°- 65°) + cos 25° cos(90°- 65°)

= sin 25° sin 25° + cos 25° cos 25°

= $\sin^2 25° + \cos^2 25°$

= 1 $[\because\ sin^2 A + cos^2 A = 1]$

Q4. सही विकल्प चुनिए और अपने विकल्प की पुष्टि कीजिए:

(i) 9 $\sec^2$ A – 9 $\tan^2$ A बराबर है:

(A) 1 (B) 9 (C) 8 (D) 0

Correct Answer: (B) 9

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Solution:

$9 \sec^2 A - 9 \tan^2 A = 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1 = 9$

(ii) (sec A + tan A) (1 – sin A) बराबर हैं :

(A) sec A (B) sin A (C) cosec A (D) cosA

Correct Answer: (D) cosA

Solution:

$(\sec A + \tan A)(1 - \sin A) = \left(\frac{1}{cosA} + \frac{sinA}{cosA}\right)(1 - \sin A)$

$= \left(\frac{1+sinA}{cosA}\right)(1 - \sin A)$

$= \frac{(1+sinA)(1-sinA)}{cosA}$

$= \frac{(1^2-sin^2A)}{cosA} = \frac{cos^2A}{cosA} = \frac{cosA \times cosA}{cosA} = \cos A$

(iii) $\frac{1+tan^2A}{1+cos^2A}$ बराबर है :

(A) $\sec^2 A$ (B) –1 (C) $\cot^2 A$ (D) $\tan^2 A$

Correct Answer: (D) $\tan^2 A$

Solution:

$$\frac{1+tan^2A}{1+cos^2A} = \frac{sec^2A}{cosec^2A}$$

$$= \frac{\frac{1}{cos^2A}}{\frac{1}{sin^2A}} = \frac{1}{cos^2A} \times \frac{sin^2A}{1}$$

$$= \frac{sin^2A}{cos^2A} = \tan^2 A$$

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Q5. निम्नलिखित सर्वसमिका सिद्ध कीजिए, जहाँ वे कोण, जिनके लिए व्यंजक परिभाषित है, न्यूनकोण है :

(i) $(\text{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

Solution:

LHS = $(\text{cosec}\theta - \cot\theta)^2$

$$= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)^2 = \frac{(1-\cos A)^2}{\sin^2 A}$$

$$= \frac{(1-\cos A)(1-\cos A)}{1-\cos^2 A}$$

$$= \frac{(1-\cos A)(1-\cos A)}{(1-\cos A)(1+\cos A)} = \frac{1-\cos\theta}{1+\cos\theta}$$

अत: LHS = RHS proved

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

Solution:

$$\text{LHS} = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{\cos^2 A + \sin^2 A + 1 + 2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{1 + 1 + 2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{2 + 2\sin A}{\cos A(1+\sin A)}$$

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$$= \frac{2\,(1+sinA)}{cosA(1+sinA)}$$

$$= \frac{2}{cosA} = 2 \times \frac{1}{cosA} = 2\text{secA}$$

(iii) $\frac{tan\theta}{1-cot\theta} + \frac{cot\theta}{1-tan\theta} = 1 + \sec\theta.\text{cosec}\theta$

Solution:

$$\text{LHS} = \frac{tan\theta}{1-cot\theta} + \frac{cot\theta}{1-tan\theta}$$

Cotθ के सभी पदों को tanθ में बदलने पर

$$= \frac{tan\theta}{1-\frac{1}{tan\theta}} + \frac{\frac{1}{tan\theta}}{1-tan\theta} = \frac{tan\theta}{\frac{\tan\theta - 1}{tan\theta}} + \frac{\frac{1}{tan\theta}}{1-tan\theta}$$

$$= \frac{tan^2\theta}{\tan\theta - 1} + \frac{1}{tan\theta(1-tan\theta)}$$

$$= \frac{tan^2\theta}{\tan\theta - 1} + \frac{1}{-tan\theta(tan\theta - 1)}$$

$$= \frac{tan^2\theta}{\tan\theta - 1} - \frac{1}{tan\theta(tan\theta - 1)}$$

$$= \frac{tan^3\theta - 1}{tan\theta(\tan\theta - 1)}$$

$$= \frac{(tan\theta-1)(tan^2\theta + tan\theta + 1)}{tan\theta(\tan\theta - 1)} \quad [\because x^3 - y^3 = (x-y)(x^2 + xy + y^2]$$

$$= \frac{(tan^2\theta + tan\theta + 1)}{tan\theta}$$

$$= \frac{tan^2\theta}{tan\theta} + \frac{tan\theta}{tan\theta} + \frac{1}{tan\theta}$$

$$= tan\theta + 1 + cot\theta$$

$$= \frac{sin\theta}{cos\theta} + 1 + \frac{cos\theta}{sin\theta} \qquad \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta} \textit{ and } \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

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$$= \frac{sin^2\theta + sin\theta.cos\theta + cos^2\theta}{cos\theta sin\theta}$$

$$= \frac{sin\theta.cos\theta + cos^2\theta + sin^2\theta}{cos\theta sin\theta} \quad [\because cos^2\theta + sin^2\theta = 1]$$

$$= \frac{sin\theta.cos\theta + 1}{cos\theta sin\theta}$$

$$= \frac{sin\theta.cos\theta}{cos\theta sin\theta} + \frac{1}{cos\theta sin\theta}$$

$$= 1 + \frac{1}{cos\theta}.\frac{1}{sin\theta} \quad \left[\because \frac{1}{cos\theta} = \boldsymbol{sec\theta},\ \frac{1}{sin\theta} = \boldsymbol{cosec\theta}\right]$$

$$= 1 + sec\theta.cosec\theta$$

Hence LHS = RHS proved

(iv) $\boldsymbol{\frac{1+secA}{secA} = \frac{sin^2A}{1-cosA}}$

Solution:

$$\text{LHS} = \frac{1+secA}{secA} = \frac{1+\frac{1}{cosA}}{\frac{1}{cosA}}$$

$$= \frac{\frac{cosA+1}{cosA}}{\frac{1}{cosA}}$$

$$= \frac{cosA+1}{cosA} \times \frac{cosA}{1} = cosA + 1$$

$$\text{RHS} = \frac{sin^2A}{1-cosA} = \frac{1-cos^2A}{1-cosA}$$

$$= \frac{(1-cosA)(1+cosA)}{1-cosA} = 1 + cosA \quad \text{या} \quad cosA + 1$$

Hence LHS = RHS

(v) $\boldsymbol{\frac{cosA - sinA + 1}{cosA + sinA - 1} = cosec\ A + cot\ A}$

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Solution:

$$\text{LHS} = \frac{cosA - sinA + 1}{cosA + sinA - 1}$$

अंश और हर को sinA से भाग देने पर

$$= \frac{\frac{cosA}{sinA} - \frac{sinA}{sinA} + \frac{1}{sinA}}{\frac{cosA}{sinA} + \frac{sinA}{sinA} - \frac{1}{sinA}} = \frac{cotA - 1 + cosecA}{cotA + 1 - cosecA}$$

$$= \frac{cotA + cosecA - 1}{cotA + 1 - cosecA}$$

$$= \frac{(cosecA + cotA) - (cosec^2A - cot^2A)}{cotA + 1 - cosecA}$$

$$= \frac{(cosecA + cotA) - (cosecA + cotA)(cosecA - cotA)}{cotA + 1 - cosecA}$$

$$= \frac{(cosecA + cotA)[1 - (cosecA - cotA)]}{cotA + 1 - cosecA}$$

$$= \frac{(cosecA + cotA)[1 - (cosecA - cotA)]}{cotA + 1 - cosecA}$$

$$= \frac{(cosecA + cotA)(1 - cosecA + cotA)}{cotA + 1 - cosecA}$$

$$= \frac{(cosecA + cotA)(cotA + 1 - cosecA)}{cotA + 1 - cosecA}$$

$$= cosecA + cotA$$

LHS = RHS

(vi) $\sqrt{\frac{1+\sin A}{1-sinA}} = \sec A + \tan A$

Solution:

$$\text{LHS} = \sqrt{\frac{1+\sin A}{1-sinA}} = \frac{\sqrt{1+\sin A}}{\sqrt{1-sinA}}$$

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हर का परिमेइकरण करने पर

$$= \frac{\sqrt{1+\sin A}}{\sqrt{1-sinA}} \times \frac{\sqrt{1+\sin A}}{\sqrt{1+sinA}}$$

$$= \frac{(\sqrt{1+\sin A})^2}{\sqrt{1-sin^2 A}} = \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

वाया पक्ष = दायाँ पक्ष

(vii) $\dfrac{sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

Solution:

$$\text{LHS} = \frac{sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{sin\theta(1 - 2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{(1 - 2(1-\cos^2\theta)}{(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{(1 - 2 + 2\cos^2\theta)}{(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{(-1 + 2\cos^2\theta)}{(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{(2\cos^2\theta - 1)}{(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta$$

वाया पक्ष = दायाँ पक्ष |

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(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

Solution:

LHS = $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + 2.\sin A.\operatorname{cosec} A + \operatorname{cosec}^2 A + \cos^2 A + 2.\cos A \,.\, \sec A + \sec^2 A$

$= \sin^2 A + 2.\sin A.\frac{1}{\sin A} + \operatorname{cosec}^2 A + \cos^2 A + 2.\cos A \,.\, \frac{1}{\cos A} + \sec^2 A$

$= \sin^2 A + 2 + \operatorname{cosec}^2 A + \cos^2 A + 2 + \sec^2 A$

$= \sin^2 A + \cos^2 A + 2 + 2 + \operatorname{cosec}^2 A + \sec^2 A$

$= 1 + 4 + (1 + \tan^2 A) + (1 + \cot^2 A)$

$= 7 + \tan^2 A + \cot^2 A$

LHS = RHS proved

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{tanA + \cot A}$

Solution:

LHS = $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$= \left(\frac{1}{\sin A} - sinA\right)\left(\frac{1}{\cos A} - cosA\right)$

$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$

$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \sin A \,.\, \cos A$

RHS = $\frac{1}{tanA + \cot A}$

$= \frac{1}{\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}}$ $\left[\because \cot\theta = \frac{\cos\theta}{sin\theta} \text{ और } \tan\theta = \frac{sin\theta}{cos\theta}\right]$

$= \frac{1}{\frac{\cos^2 A + \sin^2 A}{\sin A.\cos A}}$

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$$= \frac{1}{\frac{1}{\text{sinA.cosA}}} \quad [\because cos^2\theta + sin^2\theta = 1]$$

$$= \frac{1}{1} \times \frac{\text{sinA.cosA}}{1} = \cos A \,.\, \sin A$$

LHS = RHS proved

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

Solution:

$$\text{LHS} = \left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)$$

$$= \left(\frac{1+\tan^2 A}{1+\frac{1}{tan^2 A}}\right)$$

$$= \left(\frac{1+\tan^2 A}{\frac{tan^2 A+1}{\tan^2 A}}\right)$$

$$= \frac{1+\tan^2 A}{1} \times \frac{\tan^2 A}{1+\tan^2 A}$$

$$= \tan^2 A$$

$$\text{LHS-II} = \left(\frac{1-\tan A}{1-\cot A}\right)^2$$

$$= \left(\frac{1-\tan A}{1-\frac{1}{tan A}}\right)^2 = \left(\frac{1-\tan A}{\frac{\tan A-1}{tan A}}\right)^2$$

$$= \left(\frac{1-\tan A}{1} \times \frac{\tan A}{\tan A-1}\right)^2$$

$$= \left(\frac{1-\tan A}{1} \times \frac{\tan A}{-(1-\tan A)}\right)^2$$

$$= \left(\frac{1-\tan A}{1} \times \frac{-\tan A}{(1-\tan A)}\right)^2$$

$$= (-\tan A)^2$$

$$= \tan^2 A$$

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